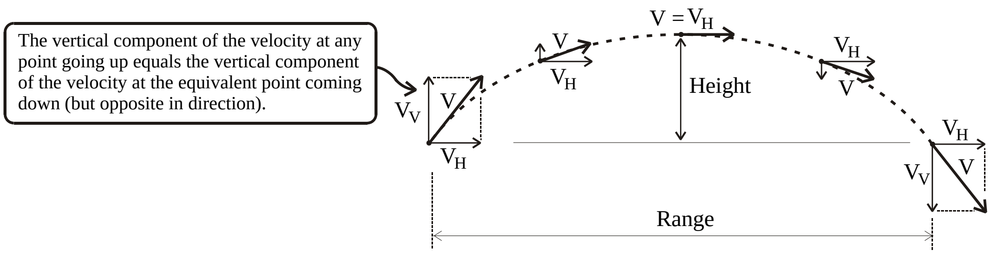
PHYSICS 534

EXERCISE-32 Projectile Motion

COMPTON

Arthur Compton received the Nobel prize for physics in 1927 for discovering the effect now known as the Compton Effect.

Projectile motion is the vectorial sum of two independent velocities, a horizontal component and a vertical component. The horizontal component velocity is ***constant*** throughout the motion while the vertical component velocity is identical to ***free fall***. The actual or instantaneous velocity at any point along the parabolic path is tangent to the parabola and equal to the vectorial sum of the horizontal and vertical component velocities.



NOTE: In solving problems on projectile motion, work out the vertical and horizontal component velocities separately as if they were two separate problems.

Use 10 m/s^2 for the acceleration due to gravity.

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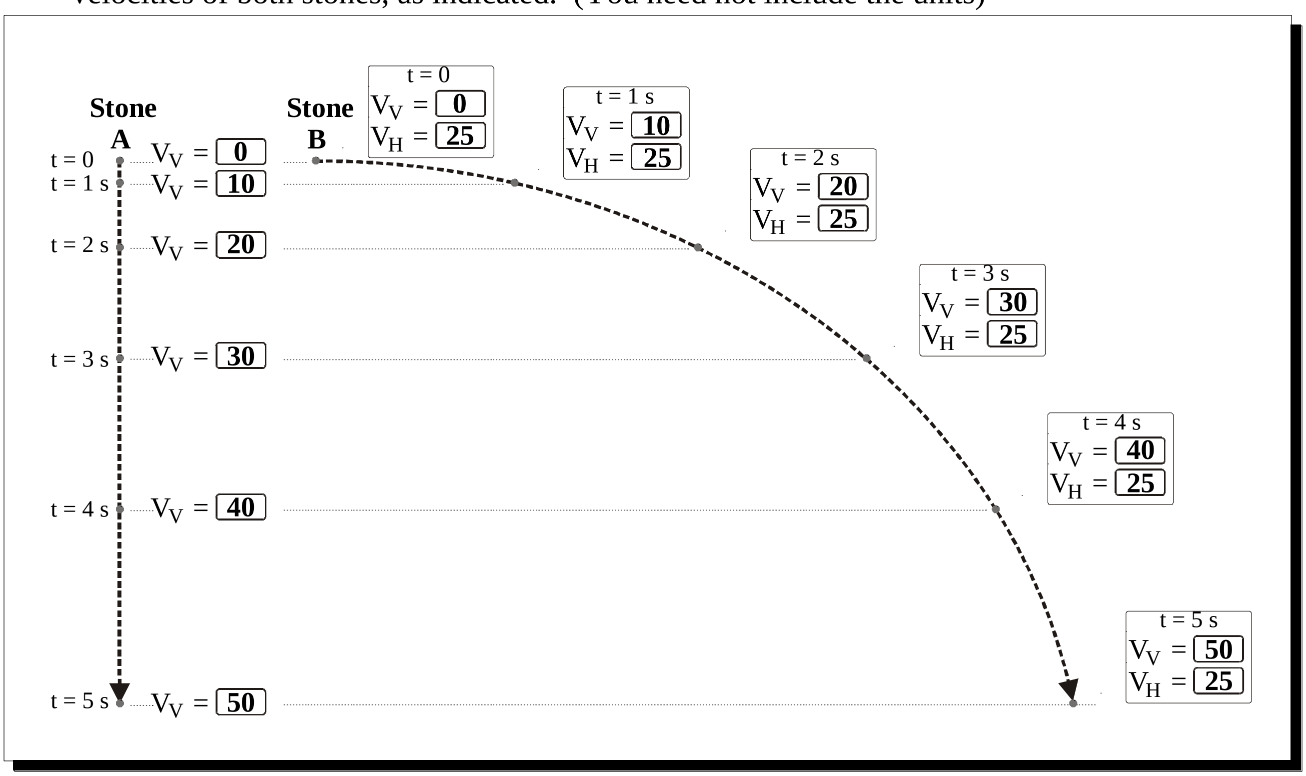
1. Two stones, A and B, are used in an experiment. At the same time that stone A is ***dropped***, stone B is thrown ***horizontally*** with a velocity of 25 m/s. For each second of fall, fill in the velocities of both stones, as indicated. (You need not include the units)



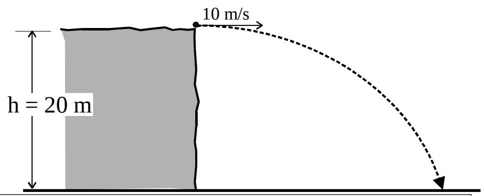
2. A stone is dropped from a balcony 20 m high. How long does it take the stone to reach the ground? [2 s]

$$2as = v_f^2 - v_i^2 \qquad a = \frac{\Delta v}{t}$$

$$v_f^2 = 2as + v_i^2 = 2(10\ m/s^2)(20\ m) = 400\ m^2/s^2 \qquad t = \frac{20\ m/s}{10\ m/s^2} = 2\ s$$

$$v_f = 20\ m/s$$

3. A ball is thrown horizontally from a 20 m high cliff with a velocity of 10 m/s. How ***far*** from the base of the cliff does the ball land? [20 m]



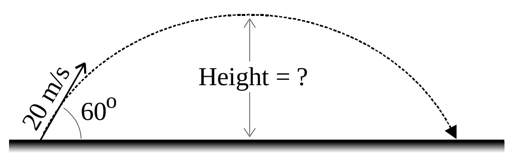
$$2as = v_f^2 - v_i^2 \qquad a = \frac{\Delta v}{t} \quad t = \frac{\Delta v}{a} = \frac{20\ m/s}{10\ m/s^2} = 2\ s$$

$$v_f^2 = 2as + v_i^2 \qquad s = v_a t$$

$$v_f^2 = 2(10\ m/s^2)(20\ m) + 0 \qquad = (\frac{20\ m/s - 0}{2})(2\ s) = 20\ m$$

$$v_f = 20\ m/s$$

4. A ball is thrown in the air with a velocity of 20 m/s at an angle of 60^o from the horizontal. Calculate:



a) How ***high*** it rises (the height). [15 m]

$$\mathbf{v_{vertical} = (20\ m/s)\sin 60^o = (20\ m/s)(0.866) = 17.3\ m/s}$$

$$\mathbf{a = \frac{\Delta v}{t} \quad t = \frac{\Delta v}{a} = \frac{v_f - v_i}{a} = \frac{0 - 17.3\ m/s}{-10\ m/s^2} = 1.73\ s}$$

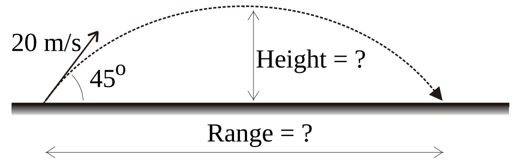
$$\mathbf{s = v_a t = (\frac{v_f + v_i}{2})t = (\frac{0 + 17.3\ m/s}{2})(1.73\ s) = 14.96\ m = 15\ m}$$

b) How ***far*** it travels (the range). [35 m]

$$\mathbf{v_{horizontal} = (20\,m/s)(Cos\,60^o) = (20\,m/s)(0.5) = 10\ m/s}$$

$$\mathbf{Range = (v_{horizontal})\,t = (10\,m/s)(1.73\,s)(2) = 34.6\ m = 35\ m}$$

5. A ball is thrown in the air with a velocity of 20 m/s at an angle of 45° from the horizontal. Calculate:



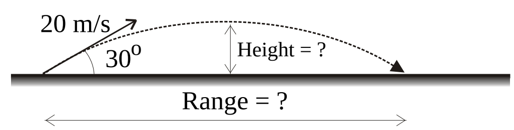
a) How ***high*** it rises (the height). [10 m]

$$v_i = 14\ m/s \quad v_f = 0 \quad a = 10\ m/s^2$$
$$\Delta v = v_f - v_i = 0 - 14\ m/s = -14\ m/s$$
$$v_a = \frac{v_f + v_i}{2} = \frac{0 + 14\ m/s}{2} = 7\ m/s$$
$$a = \frac{\Delta v}{t} \quad \text{or} \quad t = \frac{\Delta v}{a} = \frac{-14\ m/s}{-10\ m/s^2} = 1.4\ s$$
$$s = v_a t = (7\ m/s)(1.4\ s) = 9.8\ m = 10\ m$$

b) How ***far*** it travels (the range). [40 m]

$$s = v_a t = (14\ m/s)(2 \times 1.4\ s) = 39.5\ m = 40\ m$$

6. A ball is thrown in the air with a velocity of 20 m/s at an angle of 30° from the horizontal. Calculate:



a) How ***high*** it rises (the height). [5 m]

$$V_V = (20\ m/s)(\text{Sin } 30°) = 10\ m/s$$
$$a = \frac{\Delta v}{t} \quad \therefore\ t = \frac{\Delta v}{a} = \frac{v_f - v_i}{a} = \frac{0 - 10\ m/s}{-10\ m/s^2} = 1\ s$$
$$s = v_a t = \left(\frac{v_f + v_i}{2}\right)t = \left(\frac{0 + 10\ m/s}{2}\right)(1\ s) = 5\ m$$

b) How ***far*** it travels (the range). [35 m]

Since the time going up is 1 s, the total time (up and down) is 2 s.

$$V_H = (20\ m/s)(\text{Sin } 60°) = 17.3\ m/s$$
$$\text{Range} = V_H t = (17.3\ m/s)(2\ s) = 34.6\ m = 35\ m$$

7. Fill in the table below with your answers from questions 4, 5 and 6 above.

	Question	Initial velocity	Angle	Height	Range
a)	4	20 m/s	60^o	15 m	35 m
b)	5	20 m/s	45^o	10 m	40 m
c)	6	20 m/s	30^o	5 m	35 m

d) Which angle produces the greatest height? 60^o

e) Which angle produces the greatest range? 45^o

8. A cannon ball is fired into the air at an angle of 40^o from the horizontal and rises 5 m.

Find the time it takes an object to fall freely 5 m.

$s = v_i t + \frac{1}{2}at^2$ or $5\text{ m} = 0 + \frac{1}{2}(10\text{ m/s}^2)t^2$ or $t^2 = 1\text{ s}$ or $t = 1\text{ s}$

Find the vertical component of the initital velocity.

$2as = v_f^2 - v_i^2 \quad \therefore v_i^2 = v_f^2 - 2as$ or $v_i^2 = 0 - 2(-10\text{ m/s}^2)(5\text{ m})$ or $v_f = 10\text{ m/s}$

Find the horizontal component of the initial velocity.

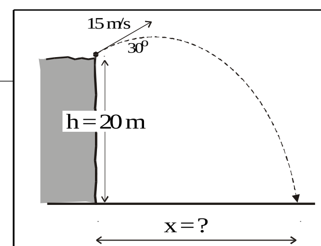
$\text{Tan } 40^o = \frac{10\text{ m/s}}{v_H}$ or $v_H = \frac{10\text{ m/s}}{\text{Tan } 40} = 11.9\text{ m/s}$

Find the range.

Time = 1 s to go up + 1 s to come down.

$\text{Range} = v_H t = (11.9\text{ m/s})(2\text{ s}) = 23.8\text{ m}$

9. A ball is thrown into the air from a cliff 20 m high.



Step – 1: Calculation of vertical and horizontal components of velocity.

$v_V = (15\text{ m/s})(\text{Sin } 30^o) = 7.5\text{ m/s}$

$v_H = (15\text{ m/s})(\text{Cos } 30^o) = 13\text{ m/s}$

Step – 2: Calculation of time to reach highest point.

$a = \frac{\Delta v}{t}$ or $t = \frac{\Delta v}{a} = \frac{v_f - v_i}{a} = \frac{0 - 7.5\text{ m/s}}{-10\text{ m/s}^2} = 0.75\text{ s}$

Step – 3: Calculation of height above cliff.

$s = v_a t = (\frac{v_f + v_i}{2})t = (\frac{0 + 7.5\text{ m/s}}{2})(0.75\text{ s}) = 2.8\text{ m}$

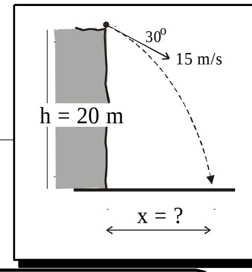
Step – 4: Calculation of time of fall from heighest point to ground (20 m + 2.8 m).

$s = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$ or $t^2 = \frac{2s}{a} = \frac{2(22.8\text{ m})}{10\text{ m/s}^2} = 4.56\text{ s}^2$ Thus: $t = 2.14\text{ s}$

Step – 5: Calculation of range.

$\text{Range} = v_H t = (13\text{m/s})(0.75\text{ s} + 2.14\text{ s}) = 37.57\text{ m} = 38\text{ m}$

10. A ball is thrown into the air from a cliff 20 m high. [redacted] m/s at an



Step – 1: Calculation of vertical and horizontal components of velocity.

$$v_V = (15)(\text{Sin } 30^o) = 7.5 \text{ m/s}$$
$$v_H = (15)(\text{Cos } 30^o) = 13 \text{ m/s}$$

Step – 2: Calculation of time of fall (20 m).

$$s = v_i t + \frac{1}{2} a t^2$$
$$\text{or } \ 20 = 7.5\,t + 5\,a t^2$$
$$\text{or } \ 5\,t^2 + 7.5\,t^2 - 20 = 0$$
$$\therefore \ t = 1.4 \text{ s}$$

Use quadratic equation:

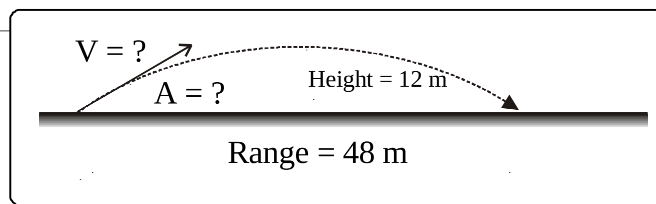
$$5t^2 + 7.5\,t - 20 = 0$$
$$A = 5 \quad B = 7.5 \quad C = -20$$
$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$= \frac{-7.5 \pm \sqrt{(7.5)^2 - 4(5)(-20)}}{2(5)}$$
$$= \frac{-7.5 \pm 21.4}{10} = \frac{-7.5 + 21.4}{10} = 1.39 \ = 1.4 \text{ s}$$

(Disregard negative value)

Step – 3: Calculation of range.

$$\text{Range} = v_H t = (13)(1.4) = 18.2 \text{ m}$$

11. A projectile is fired into the air with



Calculation of fall time (12 meters):

$$s = v_i t + \frac{1}{2} a t^2$$
$$12 \text{ m} = 0 + \frac{1}{2}(10 \text{ m/s}^2) t^2$$
$$t^2 = \frac{24 \text{ m}}{10 \text{ m/s}^2} = 2.4 \text{ s}^2$$
$$t = 1.55 \text{ s}$$

Calculation of vertical component of initial velocity (from free fall):

$$2as = v_f^2 - v_i^2$$
$$v_f^2 = 2as + v_i^2 = 2(10 \text{ m/s}^2)(12 \text{ m}) + 0 = 240 \text{ m}^2/\text{s}^2$$
$$v_f = 15.5 \text{ s}$$

Calculation of horizontal component of initial velocity (from range):

$$\text{Range} = v_H t$$
$$v_H = \frac{\text{Range}}{t} = \frac{48 \text{ m}}{2(1.55 \text{ s})} = \frac{48 \text{ m}}{3.1 \text{ s}} = 15.5 \text{ m/s}$$

Calculation of initial velocity:

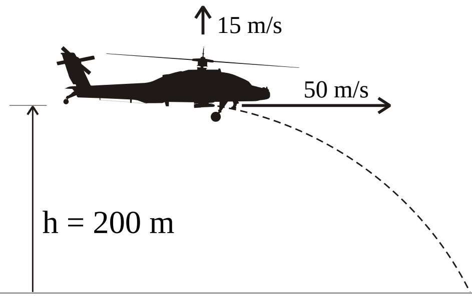
Magnitude: $v^2 = v_H^2 + v_V^2 = (15.5 \text{ m/s})^2 + (15.5 \text{ m/s})^2 = 480.5 \text{ m}^2/\text{s}^2$

$v = 21.9 \text{ m/s}$

Direction: $\text{Tan } A = \frac{v_V}{v_H} = \frac{15.5 \text{ m/s}}{15.5 \text{ m/s}} = 1 \quad \therefore \ A = 45^o$

Answer: 21.9 m/s at 45° N of E

12. A helicopter is rising vertically at 15 m/s. When it is at a height of 200 m above the ground, it fires a projectile horizontally with a velocity of 50 m/s. Determine:



a) The time it takes the projectile to hit

Note: At the instant the projectile is fired, since the helicopter is going up, it has a negative velocity.

Thus, $s = 200\ \text{m}$

$v_i = -15\ \text{m/s}$

$a = 10\ \text{m/s}^2$

$$\because \quad s = v_i t + \frac{1}{2}at^2$$

$$200 = -15t + 5t^2$$

or $5t^2 - 15t - 200 = 0$

or $t^2 - 3t - 40 = 0$

Using the quadratic equation:

$t^2 - 3t - 40 = 0$

$A = 1 \quad B = -3 \quad C = -40$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-40)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9 + 160}}{2}$$

$$= \frac{3 \pm 13}{2} = \frac{3 + 13}{2} = \frac{16}{2} = 8\ \text{s}$$

(Disregard negative value)

b) The horizontal displacement. [400 m]

Note that the helicopter is traveling at 50 m/s horizontally. Thus:

$$\mathbf{s = v_H t = (50\ m/s)(8\ s) = 400\ m}$$

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